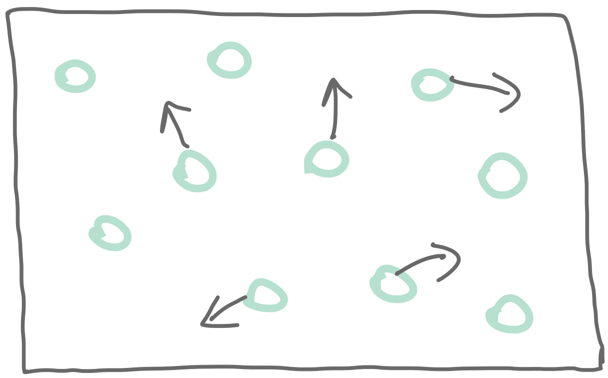
Lecture 2

Sunday, September 8, 2024 21:41

The Kinetic Model

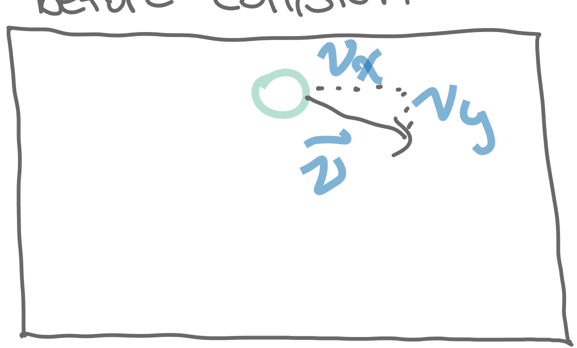


Assumptions

- particles of mass m in constant motion, obeying the laws of classical mechanics
- no interactions between the molecules (perfect gas)
- collisions are elastic (perfect transfer of energy)

Goal: derive an expression for the pressure.

before collision

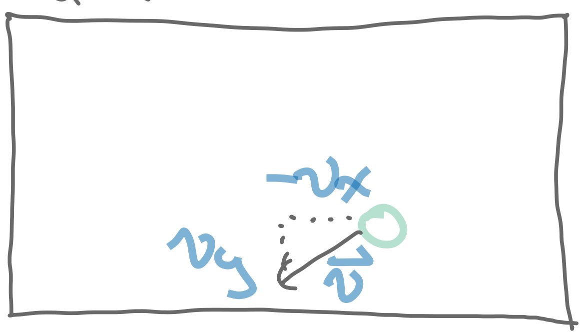


$\vec{p} = m\vec{v}$

momentum (linear)

after collision

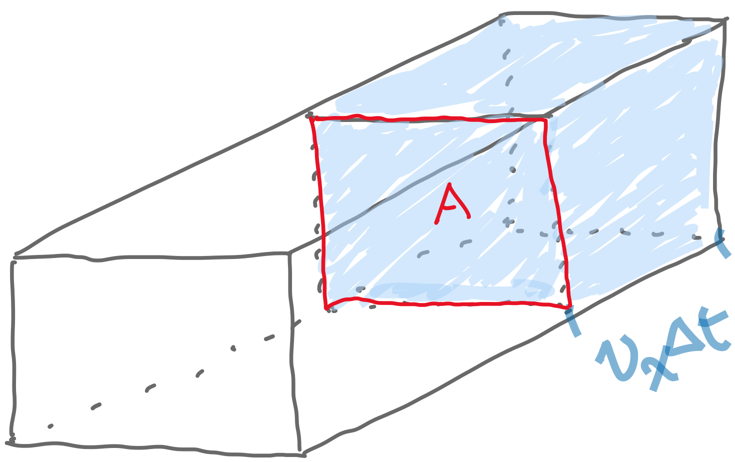
after collision



$$p_x = mv_x$$

$$\boxed{\Delta p_x = 2mv_x}$$

Next, how many collisions are there for some volume V and time Δt?



$$V = A v_x \Delta t$$

Define number density $\nu = \frac{N}{V}$

$$N = n N_A$$

The average number of collisions with the wall during time Δt is

$$\boxed{\frac{n N_A A v_x \Delta t}{2V}}$$

$m N_A = M$

$$\text{Momentum change} = \frac{n \boxed{N_A} A v_x \Delta t}{2V}$$

$\cancel{2}V$

$$\boxed{= \frac{nMAv_x^2 \Delta t}{V}}$$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$F_x = \frac{nMAv_x^2}{V}$$

$$P = \frac{nMv_x^2}{V} \longrightarrow P = \frac{nM}{}$$

$$v^2 = v_x^2 + v_y^2 + v_z^2 \qquad v_x = v$$

$$\therefore v_x^2 = \frac{1}{3} v^2$$

define root-mean-square speed

$$v_{rms} = \langle v^2 \rangle^{\frac{1}{2}}$$

$$p = \frac{1}{3} \frac{nM v_{rms}^2}{V}$$

$$\boxed{pV = \frac{1}{3} nM v_{rms}^2}$$

kinetic m
need to
RHS is e
nRT

Step 2: deriving the Maxwell-Bol distribution of speeds

$$\varepsilon = \frac{1}{2} m v^2$$

kinetic energy

Boltzmann distribution :

Boltzmann distribution:

$$\boxed{f(v) = Ke^{-\varepsilon/k_BT}}$$

$$\varepsilon = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}m$$

kinetic energy in three dime

substitute into Boltzmann distributi

$$f(v) = Ke^{-(\frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z}$$

k_B : Boltzmann's constant

$$R = N_A k_B$$

$$a^{x+y+z} = a^x a^y a^z$$

$$f(v) = Ke^{-mv_x^2/2k_BT}\, e^{-mv_y^2/2k_BT}$$

$f(v) = K e \quad e \quad e$

$f(v) = f(v_x) f(v_y) f(v_z), \quad K = k$

$$\boxed{f(v_x) = K_x e^{-m v_x^2 / 2 k_B T}}$$

need to determine K_x

$f(v_x)$ is a probability distri

which means $\int_{-\infty}^{\infty} f(v_x)\, dv_x = 1$

$$K_x \int_{-\infty}^{\infty} \underbrace{e^{-m v_x^2 / 2 k_B T}\, d v_x}_{} = 1$$

use integral table
for Gaussian function

Gaussi
fu

$$\int_0^\infty e^{-kx^2}\,dx = \frac{1}{2}\left(\frac{\pi}{k}\right)^{\frac{1}{2}} \quad k$$

$$K_x\left(\frac{2\pi k_B T}{m}\right)^{\frac{1}{2}} = 1$$

$$\therefore K_x = \left(\frac{m}{2\pi k_B T}\right)$$

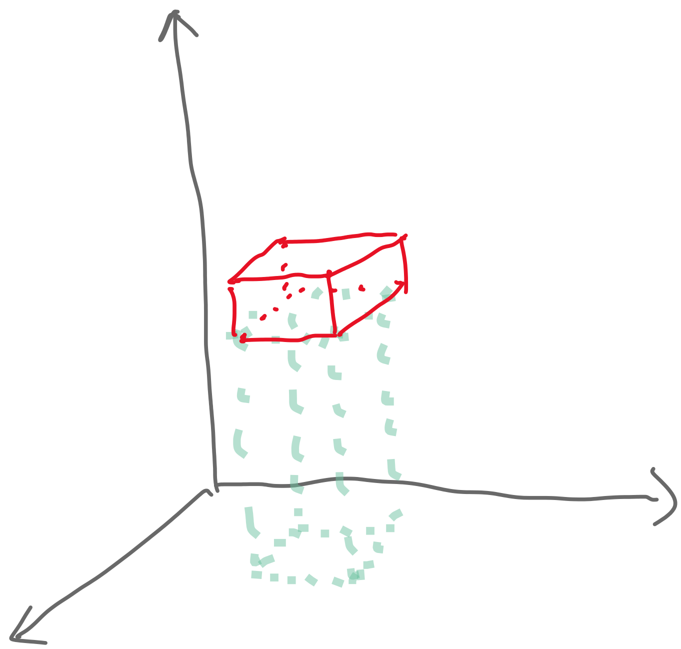
$$\boxed{f(v_x) = \left(\frac{m}{2\pi k_B T}\right)^{\frac{1}{2}} e^{-mv_x^2/2}}$$

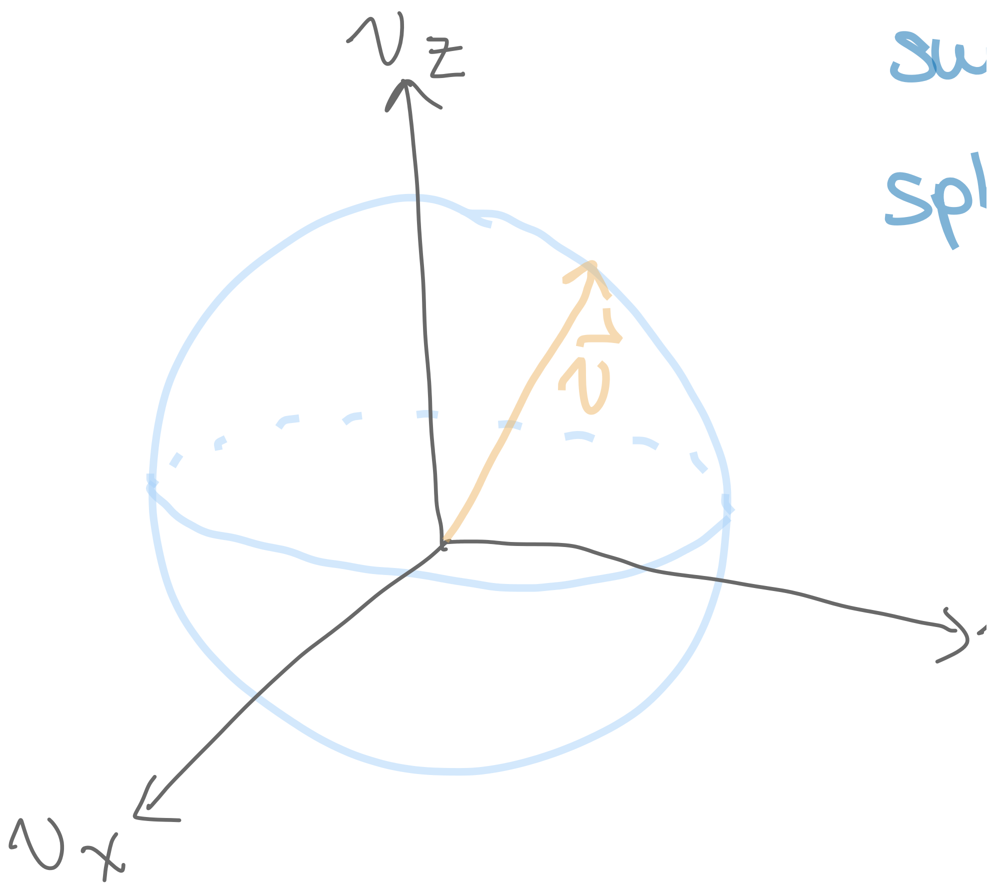
$$f(v) = f(v_x)\,f(v_y)\,f(v_z)$$

$$f(v) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} e^{-mv_x^2/2k_BT}\, e^{-mv_y^2/2k}$$

next: combine $v_x^2 + v_y^2 + v_z^2$

$$f(v_x)f(v_y)f(v_z)\,dv_x\,dv_y\,dv_z = \Big(\frac{}{2}$$





su

spl

$$f(v)\cancel{dv} = 4\pi v^2 \cancel{dv}\left(\frac{m}{2\pi k_B T}\right)^{3/2}$$

$$\boxed{f(v) = 4\pi\left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-mv^2}}$$

$$\boxed{f(v) = 4\pi\left(\frac{M}{2\pi RT}\right)^{3/2} v^2 e^{-Mv}}$$

Maxwell-Boltzmann distrib

$$\langle v^n\rangle = \int_0^\infty v^n f(v)\,dv$$

$$\langle v^2 \rangle = \frac{3RT}{M}$$

$$v_{rms} = \boxed{\left(\frac{3RT}{M}\right)^{\frac{1}{2}}}$$

recall previously that

$$pV = \frac{1}{3} n M \underline{v_{rms}}^2$$

$$pV = \frac{1}{\cancel{3}} n \cancel{M} \left(\frac{\cancel{3}RT}{\cancel{M}}\right)$$

$$\boxed{pV = nRT}$$

amazing!