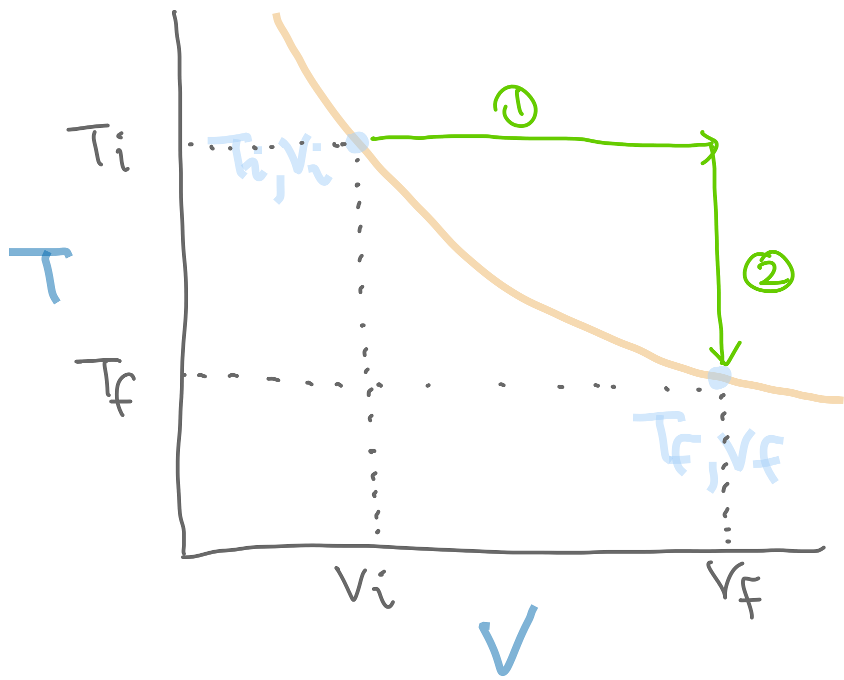
Lecture 7

Thursday, September 26, 2024 09:57

Topic 2E: Adiabatic Changes



how does the internal energy change in each of these steps?

Step 1 : $\Delta U = 0$

Step 2 : $\Delta U = C_V \Delta T$

$\Delta U = \underline{q} + w$

adiabatic ; so $q = 0$

$w_{\underline{ad}} = C_V \Delta T$

work done in an adiabatic process

$dw = \boxed{-p\,dV}$

$$-p\,dV = C_v\,dT \qquad p = \frac{nRT}{V}$$

$$-\frac{nRT}{V}\,dV = C_v\,dT$$

$$-nR\,\frac{1}{V}\,dV = C_v\,\frac{1}{T}\,dT$$

$$-nR\int_{V_i}^{V_f}\frac{1}{V}\,dV = C_v\int_{T_i}^{T_f}\frac{1}{T}\,dT$$

$$-nR\ln\frac{V_f}{V_i} = C_v\ln\frac{T_f}{T_i}$$

$$\ln\frac{V_i}{V_f} = \boxed{\frac{C_v}{nR}}\ln\frac{T_f}{T_i}$$

$$\frac{C_v}{nR} = \frac{C_{v,m}}{R} = c$$

$$\ln\frac{V_i}{V_f} = \ln\left(\frac{T_f}{T_i}\right)^c$$

$$\frac{V_i}{V_f} = \left(\frac{T_f}{T_i}\right)^c$$

$$\frac{T_f}{T_i} = \left(\frac{V_i}{V_f}\right)^{\frac{1}{c}}$$

$$\boxed{T_f = T_i \left(\frac{V_i}{V_f}\right)^{\frac{1}{c}}}$$

$$\boxed{V_i T_i^{\,c} = V_f T_f^{\,c}, \quad c = \frac{C_{V,m}}{R}}$$

Change in Pressure :

$$p_i V_i = nRT_i$$

$$p_f V_f = nRT_f$$

divide by

$$\frac{p_i V_i}{p_f V_f} = \boxed{\frac{T_i}{T_f}}$$

$$\frac{p_i V_i}{p_f V_f} = \left(\frac{V_f}{V_i}\right)^{\frac{1}{c}}$$

$$\frac{p_i}{p_f} = \boxed{\left(\frac{V_f}{V_i}\right)^{\frac{1}{c}+1}}$$

$$\boxed{\frac{p_i}{p_f}\left(\frac{V_i}{V_f}\right)^{\frac{1}{c}+1} = 1}$$

For a perfect gas, $C_{p,m} - C_{V,m} = R$

$$\frac{1}{c} + 1 = \frac{1+c}{c} = \frac{1 + \frac{C_{V,m}}{R}}{\frac{C_{V,m}}{R}}$$

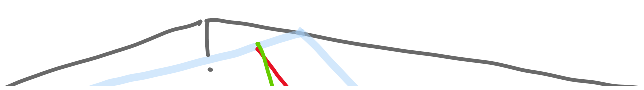
$$= \frac{R + C_{V,m}}{C_{V,m}}$$

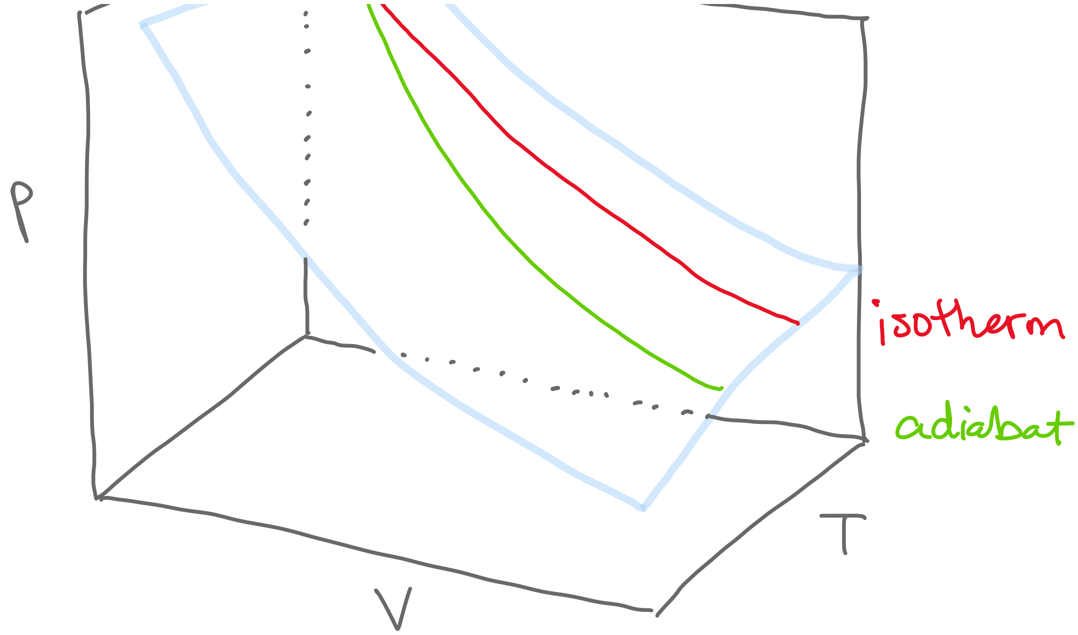
$$= \frac{C_{p,m}}{C_{V,m}}$$

Define $\gamma = \frac{C_{p,m}}{C_{V,m}}$

$$\boxed{\frac{p_i}{p_f}\left(\frac{V_i}{V_f}\right)^{\gamma} = 1}$$

$$\boxed{p_f V_f^{\gamma} = p_i V_i^{\gamma}}$$

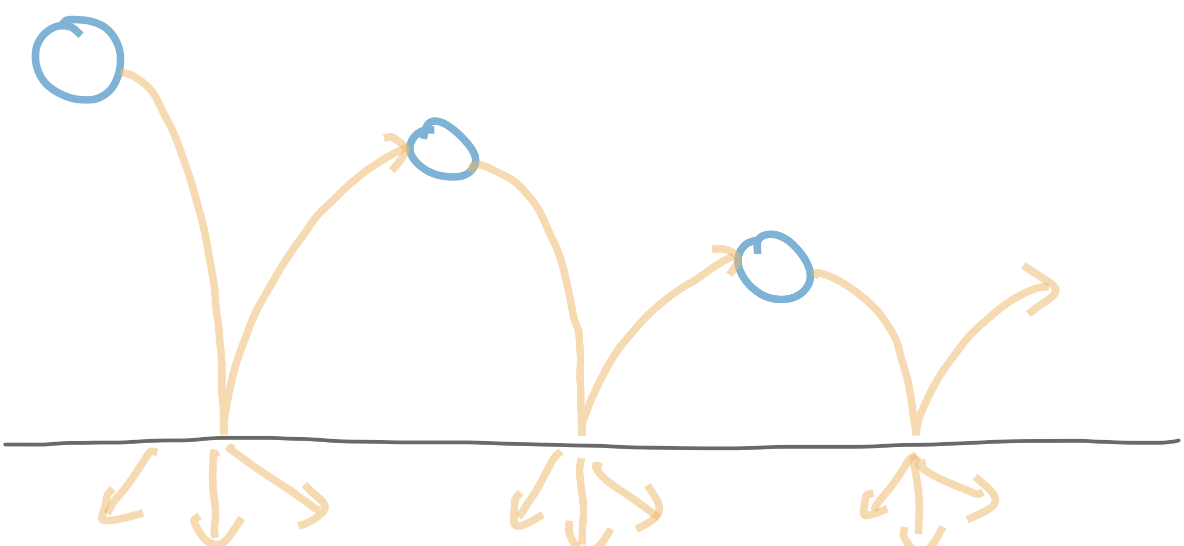




in an adiabatic process, the pressure drops more quickly compared to an isothermal process with the same initial conditions.

Topic 3A : Entropy

this is normal



this can never happen!

Analogy: states on a Rubik's cube

On a 2x2 cube, there are 3,674,160 states. But only $\boxed{1}$ is the solved state! This is similar to a scenario where all the particles in the ground move together to bounce an object upward: impossible!

more relevant for the 5x5:

2.82×10^{74} states!! Close to the number of atoms in the known universe.

Boltzmann entropy: $S = k_B \ln \Omega$

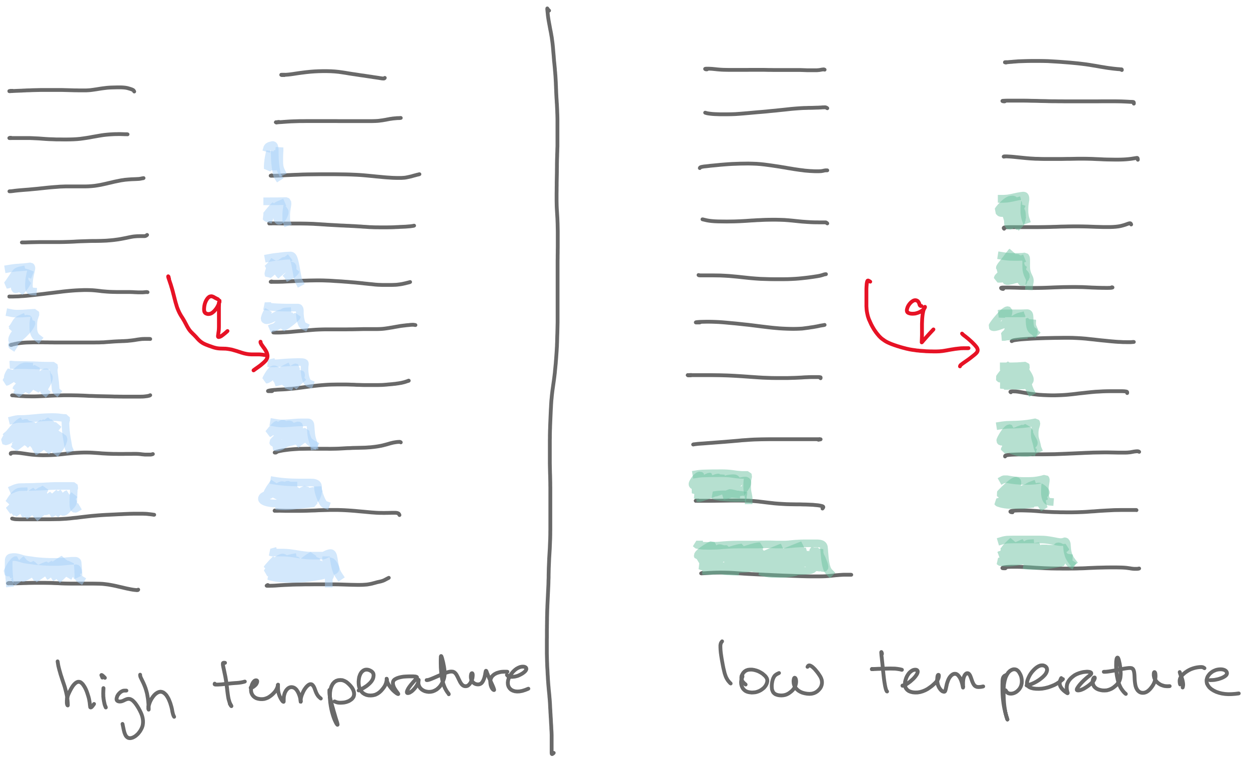
Ω: number of microstates

In thermodynamics, $dS = \frac{dq_{rev}}{T}$

q_{rev}: heat transferred reversibly

$$\Delta S = \int_i^f \frac{dq_{rev}}{T}$$

$$\boxed{\Delta S = \frac{q_{rev}}{T}}$$



$$\Delta S(\text{high } T) < \Delta S(\text{low } T)$$

The Second Law of Thermodynamics

No process is possible in which the sole result is the absorption of heat from a reservoir and its complete conversion into work.

Heat does not flow spontaneously from a cool body to a hotter body.

The entropy of an isolated system increases in the course of a spontaneous change : $\Delta S_{tot} > 0$.

$$\Delta S_{tot} = \Delta S + \Delta S_{sur}$$

1st Law : "you can't win"

2nd Law : "you can't break even"