# Lecture 2

Monday, September 09, 2024 1:56 PM

Midterm Exam: Tuesday, October 22

11:30 am, MCKN 308 (75 minutes)

**Superposition**: linear combinations of solutions to the Schrödinger equation are also solutions of the Schrödinger equation.

**Stationary state**: a quantum state with all observables independent of time.

**Wavefunction**: a mathematical description of the quantum state of an isolated quantum system.

**The Time-Independent Schrödinger Equation**

$\hbar = \frac{h}{2\pi}$ reduced Planck's

Constant

$$\underbrace{-\frac{\hbar^2}{2m}\frac{d^2\Psi(x)}{dx^2}}_{\text{kinetic energy operator}} + \underbrace{V(x)}_{\text{potential energy operator}}\Psi(x) = \underbrace{E}_{\text{energy of the system}}\Psi(x)$$

$$\underbrace{\hat{H}}_{\text{Hamiltonian}}\Psi = E\Psi \qquad \hat{H} = \hat{T} +$$

In classical mechanics,

$$E = \underbrace{T}_{\frac{p^2}{2m}} + V$$

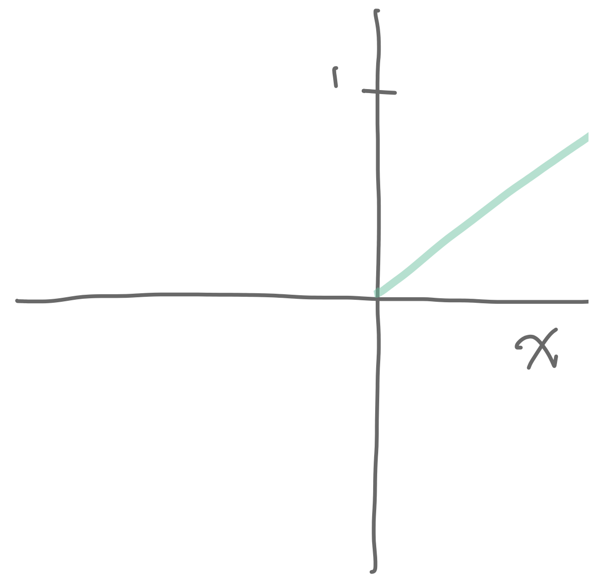
## Complex Numbers

$z = x + iy$ where $i \equiv \sqrt{\phantom{x}}$

$z = x + iy$, where ...

$\mathrm{Re}(z) = x$

$\mathrm{Im}(z) = y$




$z = 2 + i$

$|z| = (x^2 + y^2)^{\frac{1}{2}}$ complex mo

$z^* = x - iy$ complex conjug

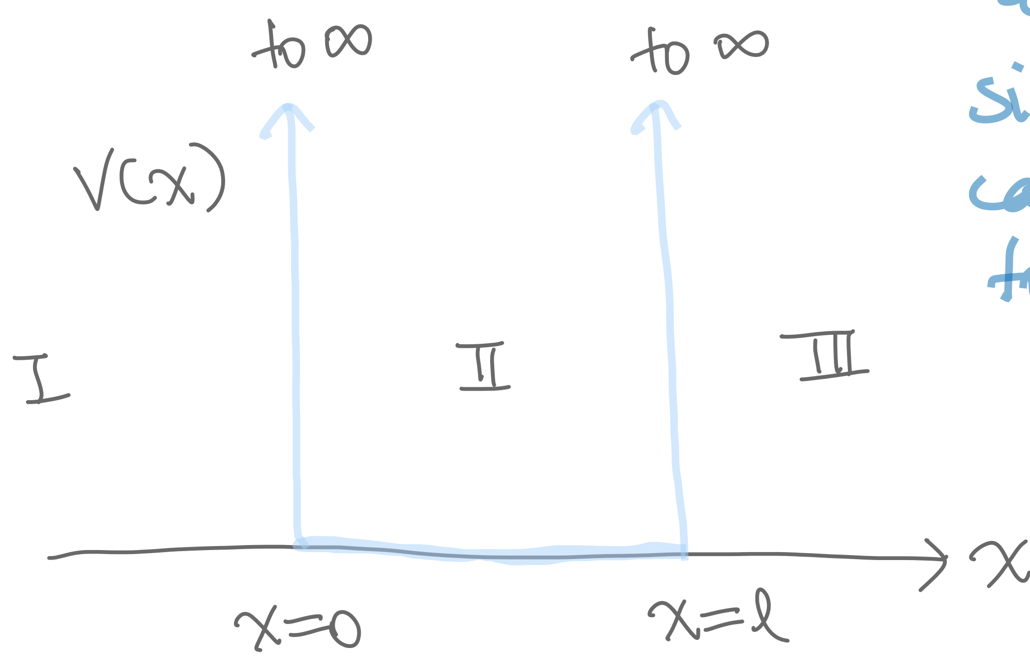
$$zz^* = (x+iy)(x-iy)$$

$$= x^2 - \cancel{ixy} + \cancel{ixy} - \overset{-1}{\underline{i^2}}\, y^2$$

$$\boxed{zz^* = x^2 + y^2 = |z|^2}$$

$\Psi^*\Psi = |\Psi|^2$ probability d

## The Particle in a Box




## Differential Equations

ordinary : one independent varial

(as opposed to partial

linear : terms are only r
the first power

homogeneous : RHS is zero

e.g. order

$$y^{\boxed{'''}} + 2x(y')^2 + y^2 \sin x$$

we restrict ourselves to ordinary
homogeneous differential equation

$$\boxed{y'' + P(x)y' + Q(x)y = 0}$$

Suppose that $y_1$ and $y_2$ ar
independent functions , which so

equation above. The general solu

$$\boxed{y = c_1 y_1 + c_2 y_2}$$

$$c_1 y_1'' + c_2 y_2'' + P(x) c_1 y_1' + P$$

$$+ Q(x) c_1 y_1 + Q(x) c_2 y_2$$

$$= c_1 \underbrace{[y_1'' + P(x) y_1' + Q(x) y_1]}_{0} + c_2 [y_2'' +$$

$$= 0$$

Add in the condition of constant

$$y'' + p y' + q y = 0 \quad , \; p, q$$

try $y = \underline{e^{ax}}$

$$\boxed{a^2 e^{ax} + pae^{ax} + qe^{ax} =}$$

$$\boxed{a^2 + pa + q = 0}$$

a

qu

tw

$a_1$

$$\boxed{y = c_1 e^{a_1 x} + c_2 e^{a_2 x}}$$

Back to the PIAB. In region

$V = \infty$, so

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \infty\psi = E\psi$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = (E-\infty)\psi$$

$$\cancel{-}\cancel{\frac{\hbar^2}{2m}}\frac{d^2\psi}{dx^2} = \cancel{-}\infty\psi$$

$$\frac{1}{\infty}\frac{d^2\psi}{dx^2} = \psi$$

$$\psi_{I} = 0 \quad \text{and} \quad \psi_{III} = 0$$

The particle wavefunction is ze
the box.

For region II,

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_{II}}{dx^2} + \cancel{0\psi_{II}} = E\psi_{II}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_{II}}{dx^2} - E\psi_{II} = 0$$

$$\boxed{\frac{d^2\psi_{II}}{dx^2} + \frac{2mE}{\hbar^2}\psi_{II} = 0}$$

$$y'' + \frac{2mE}{\hbar^2}y = 0$$

apply the auxiliary equation

$$a^2 + 2mE\hbar^{-2} = 0$$

$$a^2 = -2mE\hbar^{-2}$$

$$a = \pm(2mE)^{\frac{1}{2}}\hbar^{-}$$

$$\psi_{II} = c_1 e^{-(2mE)^{\frac{1}{2}} ix/\hbar} + c_2 e^{+}$$

$$\text{Let } \theta \equiv \frac{(2mE)^{\frac{1}{2}} x}{\hbar}$$

$$\boxed{\psi_{II} = c_1 e^{i\theta} + c_2 e^{-i\theta}}$$

$$\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$$

$$e^{-i\theta} = \cos(-\theta) + i\sin(-\theta)$$

$$\boxed{e^{-i\theta} = \cos\theta - i\sin\theta}$$

$$\psi_{II} = c_1\cos\theta + ic_1\sin\theta + c_2\cos\theta$$

$$= \underbrace{(c_1 + c_2)}_{A}\cos\theta + \underbrace{i(c_1 - c_2)}_{B}$$

$$\boxed{\psi_{II} = A\cos\theta + B\sin\theta}$$

$$\therefore \psi_{II} = A\cos\left[\frac{(2mE)^{\frac{1}{2}}}{\hbar}x\right] + B\sin\left[\frac{(}{-}\right.$$