## Lecture 5

Tuesday, September 24, 2024 11:28

Practice Problem  
When a particle of mass 9.1×10<sup>-28</sup> g  
in a certain one-dimensional box goes  
from the n=s level to the n=2  
level, it emits a photon of frequency  

$$6.0 \times 10^{14} \text{ s}^{-1}$$
. Find the length of the  
box.  
 $hv = (n_{upper}^2 - n_{lower}^2) \frac{h^2}{8ml^2}$   
 $l^2 = \frac{21h}{8mv}$   
 $l = (\frac{21}{8} \frac{6.626 \times 10^{-34} \text{ kg} \cdot \text{m}^2 \cdot \text{s}^{-1}}{9.1 \times 10^{-31} \text{ K} \cdot 6.0 \times 10^{14} \text{ s}^{-1}})$   
 $l = 1.78 \times 10^{-9} \text{ m} < round to two significant figures}$ 

Operators Continued

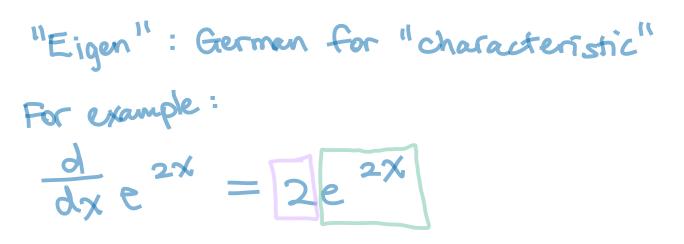
Distributive identifies  $(\hat{A} + \hat{B})\hat{C} = \hat{A}\hat{C} + \hat{B}\hat{C} \leftarrow \text{prove this}$  $\hat{A}(\hat{B}+\hat{C}) = \hat{A}\hat{B} + \hat{A}\hat{C}$  $\left[\left(\hat{A} + \hat{B}\right)\hat{c}\right]f = \left(\hat{A}\hat{c} + \hat{B}\hat{c}\right)f$  $\left[ (\hat{A} + \hat{B}) \hat{c} \right] f = (\hat{A} + \hat{B}) (\hat{c} f)$ result is Definition from the last lecture:  $(\hat{A} + \hat{B}) f(x) \equiv \hat{A} f(x) + \hat{B} f(x)$  $\left[\left(\hat{A} + \hat{B}\right)\hat{C}\right]f = \hat{A}\left(\hat{C}f\right) + \hat{B}\left(\hat{C}f\right)$ Another definition from the last lecture :  $\hat{A}\hat{B}f(x) \equiv \hat{A}\hat{B}f(x)$ 

$$\begin{split} [(\hat{A} + \hat{B})\hat{C}]f &= \hat{A}\hat{C}f + \hat{B}\hat{C}f\\ [(\hat{A} + \hat{B})\hat{C}]f &= (\hat{A}\hat{C} + \hat{B}\hat{C})f\\ \therefore (\hat{A} + \hat{B})\hat{C} &= \hat{A}\hat{C} + \hat{B}\hat{C} \\ \hline \therefore (\hat{A} + \hat{B})\hat{C} &= \hat{A}\hat{C} + \hat{B}\hat{C} \\ \hline Find the square of the operator \\ \frac{d}{dx} + \hat{x} \\ Define \hat{D} &\equiv \frac{d}{dx}; apply (\hat{D} + \hat{x})^2 \\ to an arbitrary function f(x). \\ (\hat{D} + \hat{x})^2 f(x) &= (\hat{D} + \hat{x})[\hat{D} + \hat{x}]f \\ &= (\hat{D}^2 + \hat{x})(f' + xf) \\ \hat{x} &= x^{-1} \\ &= f^{11} + f + xf' + xf' + x^2f \\ &= (\hat{D}^2 + 2\hat{x}\hat{D} + \hat{x}^2 + 1)f(x) \\ (\hat{D} + \hat{x})^2 &= \hat{D}^2 + 2\hat{x}\hat{D} + \hat{x}^2 + 1 \end{split}$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

3.2 Eigenfunctions and Eigenvalues  
eigenvalue  

$$\hat{A} f(x) = \hat{k} f(x)$$
  
eigenfunction



If f(x) is an eigenfunction of the linear operator  $\hat{A}$  and C is any constant, prove that cf(x) is an eigenfunction of  $\hat{A}$  with the same eigenvalue as f(x). Definition for linear operators from the last lecture :

$$A(bt) = bAt$$

counterexample : 
$$sin(2x) \neq 2sin(x)$$
  
Need to prove  $\hat{A}(cf) = k(cf)$   
 $\hat{A}(cf) = c\hat{A}f = ckf = k(cf)$ 

$$\therefore \hat{A}(cf) = k(cf)$$

a) Find the eigenfunctions and eigenvalues  
of the operator 
$$\frac{d}{dx}$$
.  
 $\hat{A} = kf(x)$ 

$$\frac{d f(x)}{dx} = k f(x)$$

$$\frac{f(x)}{f(x)} df(x) = K dx$$

$$f(x) df(x) = k \int dx$$
  

$$ln f(x) = kx + constant$$
  

$$f(x) = e^{kx + constant}$$
  

$$f(x) = e^{kx}$$

b) If we impose the boundary condition that the eigenfunctions remain finite as  $x \to \pm \infty$ , find the eigenvalues.

$$k = a + ib$$

$$f(x) = ce^{ax} e^{ibx}$$
If  $a < 0$ , then  $f(x) \rightarrow \infty$  as  $x \rightarrow -\infty$ 
If  $a > 0$ , then  $f(x) \rightarrow \infty$  as  $x \rightarrow \infty$ 

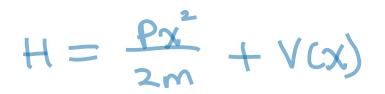
$$\therefore a = 0$$
,  $k = ib$ 
Commutators Revisited

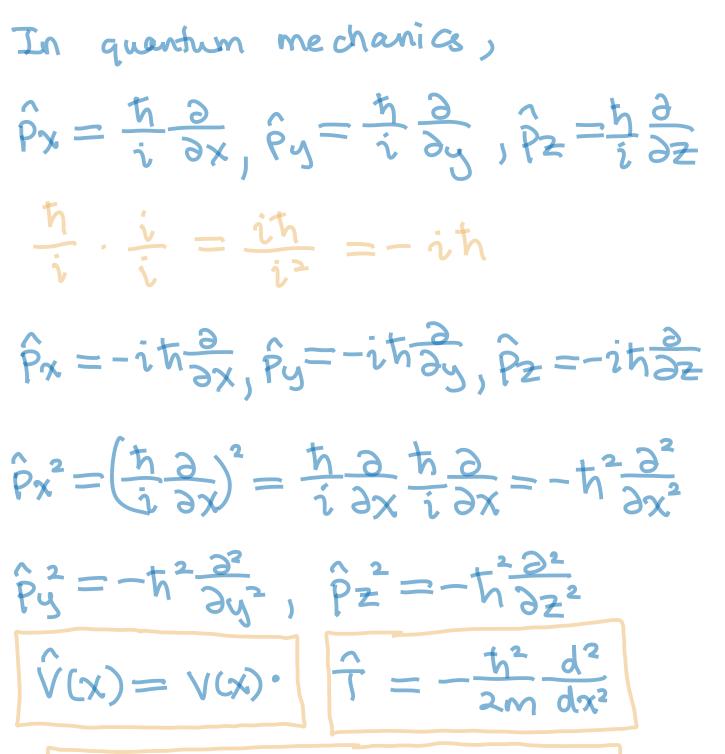
https://teams.microsoft.com/v2/

	a Rul	ak's c form	voe,	a commutator
R	L	RI	L'	Jorja
R	U	R'	U <sup>1</sup>	Kate
٢	R	۲'	R'	Thomas
Operations that overlap completely or do not averlap at all commute;				
operations that partially overlap do Not commute.				

3.3 Operators and Quantum Mechanics Linear momentum in classical mechanics  $Px \equiv mv_x$ ,  $Py \equiv mv_y$ ,  $Pz \equiv mv_z$  $V_x = \frac{Px}{V_x}$ ,  $T = \pm mv_z^2$  Teams and Channels | General | University of Guelph | lchen22@uoguelph.ca | Microsoft Teams

Z VIVX





 $\hat{H} = \hat{T} + \hat{V} = -\frac{\hbar^2}{2} \frac{d^2}{dx^2} + V(x)$ 

. .

LY

Is Y(x) for the one-dimensional PIAB an eigenfunction of Px?  $\Psi(x) = \left(\frac{2}{\ell}\right)^{\frac{1}{2}} \sin\left(\frac{n\pi x}{\ell}\right)$  $\hat{p}_{x}\Psi(x) = -i\hbar\frac{d}{dx}\left[\left(\frac{2}{l}\right)^{\frac{1}{2}}\sin\left(\frac{n\pi x}{l}\right)\right]$  $\hat{\rho}_{\chi}\Psi(\chi) = -\frac{n\pi i\hbar}{\ell} \left(\frac{2}{\ell}\right)^{\frac{1}{2}} \cos\left(\frac{n\pi \chi}{\ell}\right)$  $\hat{p}_{x}\Psi(x) \neq k\Psi(x)$ .: Y(x) for PIAB is not an eigenfunction of  $\beta x$ . Despite this, whenever we make measurement for px, we must still get one of the eigenvalues for  $\hat{p}_{\chi}$ .

 $\hat{\rho}_{x}^{2}\varphi(x) = -\hbar^{2}\frac{d^{2}}{dx^{2}}\left[\left(\frac{2}{p}\right)^{\frac{1}{2}}\sin\left(\frac{n\pi\chi}{p}\right)\right]$ https://teams.microsoft.com/v2/

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$$= \frac{n^2 \pi^2 h^2}{\ell^2} \left(\frac{2}{\ell}\right)^{\frac{1}{2}} \sin\left(\frac{n\pi \chi}{\ell}\right)$$

$$\therefore \hat{p}_{x}^{2} \Psi(x) = \frac{n^{2} h^{2}}{4 \ell^{2}} \Psi(x)$$

$$E = \frac{n^2 h^2}{8ml^2}$$

Y(x) is an eigenfunction of  $\hat{p}_x^2$ . Notation for a three-dimensional, many-porticle system:

Laplacian operator, 
$$\nabla^2$$

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

One particle in three dimensions:

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + V(x,y,z)\Psi = E\Psi$$

n-particles in three dimensions:  

$$\hat{T} = -\frac{\hbar^2}{2m_1} \left( \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial y_1^2} + \frac{\partial^2}{\partial z_1^2} \right)$$
particle i  

$$-\frac{\hbar^2}{2m_2} \left( \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial y_2^2} + \frac{\partial^2}{\partial z_2^2} \right)$$
particle 2  

$$-\frac{\hbar^2}{2m_1} \left( \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial y_2^2} + \frac{\partial^2}{\partial z_2^2} \right)$$
particle 2  

$$-\frac{\hbar^2}{2m_1} \left( \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial y_1^2} + \frac{\partial^2}{\partial z_1^2} \right)$$
particle n  

$$\hat{T} = -\sum_{i=1}^{n} \frac{\hbar^2}{2m_i} \nabla_i^2$$

 $\sum_{i=2m_{i}}^{n} \nabla_{i}^{2} + V(x_{i}, y_{i}, \dots z_{n})$